Whole Number Addition & Subtraction Progressions K-5

For progressions of addition and subtraction with fractions and decimals, see the Fractions Progressions K-5.

	Understanding the Operations	Strategies & Properties	Fluency

K	• Move from subitizing to conceptual subitizing, i.e. I see three dots because there are two here and one more there (**see Algebraic Progressions**)
	• Extend counting strategies to represent, model, and solve simple addition and subtraction situations (**see Number Sense Progressions**)
	• Represent addition and subtraction situations
	  ○ Model using objects, fingers, mental images, drawings, sounds, or acting out
	  ○ Connect to verbal explanations
	  ○ Connect to expressions and equations given by the teacher
	  ○ Model how to put together (compose) or take apart (decompose) up to 10 objects in different ways
	• Solve simple addition and subtraction word problems within 10
	  ○ Use objects or drawings to represent the problem
	  ○ Represent, model, and solve the following problem types:
	    ■ Add to – Result Unknown 8 + 3 = ?
	    ■ Take from – Result Unknown 7 – 2 = ?
	    ■ Put Together – Total Unknown 5 + 3 = ?
	    ■ Take Apart – Both Addends Unknown 6 = ? + ? (in both addends unknown problems, also include matching with equations using symbols to represent the unknown)
	• For any number 1 to 9, find the number that makes 10 when added (missing addend)
	  ○ Use physical models and drawings to represent ways 10 can be composed
	  ○ Match representations to equations and expressions given by the teacher
		• For any number 1 to 9, find the number that makes 10 when added (missing addend)
		  ○ Use a variety of strategies to compose 10, e.g. number bonds, physical objects, part-part-whole chart, number strips, five-frames, ten-frames, etc.
		  ○ Understand the numbers can be put together and taken apart in different ways
		  ○ Work with compositions of 5 before moving to compositions of 10
		  ○ Begin to develop simple strategies such as doubles facts, counting on 1 or 2, adding zero, and decomposing an addend (i.e. 2 + 3 can be thought of as 2 + 2 + 1)
		• Students in kindergarten mainly work at at a Level 1 sophistication for solving addition and subtraction problems
		  ○ Represent situations with groups of objects, a drawing, or by fingers
		  ○ Adding All for addition: represent each addend separately with objects, put the two groups together, then count all objects
		  ○ Adding All for subtraction: represent the total amount of objects, take the known addend number of objects away, count the remaining objects
		  ○ Some students may be ready to progress to next levels, see Grade 1 for Level 2 and Level 3 strategies
			• Fluently add and subtract within 5
			  ○ Understand what it means to add or subtract
			  ○ Give a sum or difference in about 3 seconds without resorting to counting (counting on fingers does not constitute fluency of facts)
			  ○ Solve problems involving sums to 5 through concrete models and drawings

1	• Solve addition and subtraction word problems within 20
	  ○ Represent and use objects, drawings or other strategies to solve
	  ○ Write equations, including using a symbol to represent the unknown, to represent their work
	  ○ Identify what they are looking for in the context of the problem, e.g. the total number of rabbits
	  ○ Extend work from kindergarten problem types to now include:
		• Solve addition and subtraction word problems within 20
		  ○ Use and explain meaningful strategies including counting up, counting back, making groups of 10, and finding missing addends
		• Extend strategies from Level 1 to begin to work with Level 2 and 3 strategies
		  ○ Level 2 includes counting on to find to total or find the addend
		  ○ Counting on in adding: Count on from the larger addend, counting stops
			• Demonstrate fluency for addition and subtraction within 10
			  ○ Explain their strategy for finding the answer to an addition or subtraction fact with sums to 10
			  ○ Justify strategies based on efficiency and number sense
			  ○ Extend use of strategies to facts with sums to 20,

- Add to – change unknown
- Add to – start unknown
- Take from – change unknown
- Take from – start unknown
- Put together/Take apart – addend unknown
- Compare – difference unknown
- Compare – bigger unknown
- Compare – smaller unknown

- Solve word problems with addition of three whole numbers within 20
  o Represent and use objects, drawings or other strategies to solve
  o Write equations, including using a symbol to represent the unknown, to represent their work
  o Extend and apply meaningful strategies from work with two addends
- Understand subtraction as an unknown-addend problem
  o Identify parts of addition and subtraction equations using the terms addend, missing addend, and total
  o Use this inverse relationship between addition and subtraction to practice fact fluency
- Determine if equations involving addition and subtraction are true or false (**see Algebraic Progressions**)
- Determine the unknown whole number in an addition or subtraction equation relating three whole numbers (**see Algebraic Progressions**)
- Add within 100
  o Add a two-digit number and a one-digit number
    - First, without the need to compose a new ten (42 + 5)
    - Then, with the need to compose a new ten (46 + 7)
  o Add a two-digit number and a two-digit number
    - First, without need to compose a new ten (42 + 35)
    - Then, with the need to compose a new ten (34 + 28)
  o Add a two-digit number and a multiple of 10 (52 + 30)
  o Model using concrete materials, pictures, and numerals
  o Recording addition problems both horizontally and vertically

when the second addend has been counted, last number said is the total
o Counting on to find unknown addend: Count on from the known addend, stop when the known total has been reached, keep track of counts to reach total, understand the count is the missing addend
o Counting on to subtract: Thinks of subtracting as finding the unknown addend, uses above strategy based on knowledge of inverse relationship of addition and subtraction
o Level 3 strategies include Converting to an Easier Equivalent Problems using distributive, commutative, associative properties and knowledge of inverse operations
o Adding by making a ten: decompose one addend to make a number value which when added to the first addend will make ten, combine the ten and remaining value to find the total (8 + 3 is the same as 8 + 2 + 1)
o Adding using doubles plus/minus 1: Knowing a double fact, recognizing two addends are only 1 count away from a doubles fact and using that knowledge to find the total (6 + 7 is the same as 6 + 6 + 1)
o Finding an unknown addend by making a ten: for 8 + ? = 14, thinking 8 + 2, and then 4 more makes 14, the missing addend is 6
o Finding an unknown addend using doubles plus/minus 1: for 6 + ? = 13, thinking double 6 is 12, and 1 more is 13, the missing addend is 6 and 1 (7)
o Apply above strategies to use missing addend to subtract: Thinks of subtracting as finding the unknown addend, uses above strategies based on knowledge of inverse relationship of addition and subtraction
o Finally students apply these strategies to re-represent Add to – start unknown problems as missing addend problems (? + 6 = 8 is the same as 6 + ? = 8), and re-represent Take from – start unknown problems as missing total problems (? – 6 = 8 is the same as 8 + 6 = ?)

using objects, drawings, and symbolic representations

	o Make generalizations and develop their own strategies for adding one- and two-digit numbers o Use models to represent and reason when and why it is appropriate to compose a new ten o Make justifications based on place value understanding **DO NOT need to use the standard algorithms** • Given a two-digit number, mentally find 10 more or 10 less, without having to count o Explain their reasoning using place value understanding and patterns o Use variety of materials to model and reason, including a hundred chart • Use place value reasoning to subtract multiples of 10 from multiples of 10 (e.g. 70 – 30) o Model with a variety of materials, including physical objects, pictures, and words o Explain their reasoning using place value understanding and patterns on the hundred chart o Calculate using both mental strategies and using written equations	• Recognize the commutative and associative properties can be applied to add three whole numbers o Understand the order of the addends does not change the value, and changing the order could make some problems easier to solve, e.g. to solve 2 + 6 + 4, it may be easier to add 6 + 4 first to make 10 o Describe patterns and make generalizations in regards to strategies applied • Mentally calculate to find 10 more or 10 less than a given number	
2	• Add and subtract within 100 to solve one- and two-step word problems o Use objects, drawings, and equations to represent the context o Write equations, including the use of a symbol to represent the unknown number in the problem o Connect all problem types from 1st grade to two-step word problems o Solve using a variety of strategies, including concrete and pictorial representations, and explain their reasoning o Explain and justify answers verbally and/or in writing • Connect double facts to find odd and even groups of objects (**see Algebraic Progressions**) • Connect repeated addition of equal rows/columns to find number of objects in arrays (**see Algebraic Progressions**) • Fluently add and subtract within 100 o Extend previous work with place value and physical models	• Add and subtract within 100 to solve one- and two-step word problems o Solve using a variety of strategies, including concrete and pictorial representations, e.g. part-part-whole, number lines, drawings, ten frames, cubes, base-ten blocks, etc. o Explain and justify answers verbally and/or in writing • Add up to four two-digit numbers using strategies based on place value o Apply the associative property to solve, e.g. to solve 24 + 17 + 33, may add 17 and 33 to get 50, then 50 and 24 to get 74 o Apply place value understanding, e.g. to solve 24 + 17 + 33, add all the tens, 20 + 10 + 30 = 60, then add the ones, 7 + 3 + 4 = 14, then combine the two sums (60 and 14) to get 74 • Add and subtract within 1000 o Use concrete models relating to place value o Use drawings or strategies based on place value	• Fluently add and subtract within 20 o Use mental math strategies o Apply and extend strategies from facts within 10 o Know "from memory" all sums of two one-digit numbers (3 seconds per fact) • Fluently add and subtract within 100 o Extend previous work with place value and physical models o Use place value understanding to compose and decompose tens o Solve equations both horizontally and vertically o Explain and justify answers verbally and/or in writing

	o Use variety of strategies including physical models, mental computation, benchmark numbers, place value charts, number lines, and hundred charts
	o Justify and reason with others about their strategies
	o **DO NOT need to use the standard algorithms**
	• Add up to four two-digit numbers using strategies based on place value
	• Add and subtract within 1000
	  o Extend previous work with place value and physical models
	  o Use variety of strategies including physical models, mental computation, benchmark numbers, place value charts, number lines, and hundred charts
	  o Justify and reason with others about their strategies
	  o **DO NOT need to use the standard algorithms**
	• Extend work from 1st grade to mentally add or subtract 10 or 100 to a given number within 1000
	  o Use a variety of strategies to add or subtract 10 and 100
	  o Explain their reasoning using place value understanding and patterns
	• Use addition and subtraction within 100 to solve word problems involving lengths (**see Measurement Progressions**)
		o Apply properties of operations and relationship between addition and subtraction
		o Relate strategies to a written method
		o Understand that when adding three-digit numbers, one adds hundreds and hundreds, tens and tens, and ones and ones
		o Understand that when subtracting three-digit numbers, one subtracts hundreds and hundreds, tens and tens, and ones and ones
		o Sometimes when adding or subtracting, one must compose or decompose tens and hundreds
		• Extend work from 1st grade to mentally add or subtract 10 or 100 to a given number within 1000
		  o Use mental strategies and patterns to make generalizations	

3	• Solve two-step word problems using the four operations
	  o Represent and solve these problems using models, pictures, words and equations
	  o Justify reasonableness of answers using estimation strategies
	• Solve word problems involving addition and subtraction with time intervals (**see Measurement Progressions**)
	• Solve problems involving masses or volumes, using the four operations (**see Measurement Progressions**)
	• Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs (**see Data Progressions**)
	• Solve real world problems involving perimeter of polygons (**see Measurement Progressions**)
		•	• Fluently add and subtract within 1000
			  o Use strategies to begin to develop algorithms based on place value understanding
			  o Use a variety of models, representations, and strategies
			  o Write equations both vertically and horizontally
			  o Make connections between conceptual understanding and procedures

	• Recognize area as additive, apply addition to solve problems involving the area of rectilinear figures (**see Measurement Progressions**)		

4	• Solve multistep (up to 3-steps) word problems with whole numbers, using the four operations
	  o Use and explain their problem-solving process using models, or pictures or numbers and words
	  o Add and subtract within 1000
	• Determine the unknown whole number in equations involving four numbers using comparative relational thinking and determine if equations are true or false (**see Algebraic Progressions**)
	• Given a rule, generate or extend a number or shape pattern (**see Algebraic Progressions**)
	• Solve word problems involving distances, time intervals, liquid volumes, masses of objects and money, using the four operations (**see Measurement Progressions**)
	• Apply the perimeter formula, with both real world and mathematical problems (**see Measurement Progressions**)
	• Recognize angle measure as additive, solve problems involving missing angle measurements (**see Measurement Progressions**)
	• For addition and subtraction work with fractions, (**see the Fractions Progressions**)
		•	• Fluently add and subtract multi-digit whole numbers (factors and solutions between 1,000 and 1,000,000)
			  o Use efficient mental strategies to compute when appropriate
			  o Use the standard algorithms to add and subtracts
			  o Make connections between strategies with addition and subtraction to developing an efficient algorithm
			  o Explain their thinking as they employ procedural steps to add and subtract
			  o Add up to three addends

5	• Extend whole number work with addition and subtraction to situations involving fractions and decimals, including work with all problem situations (**see Fractions Progressions**)	•	•