Algebraic Thinking Progression K-5

Counting and Cardinality, Operations, and Algebraic Thinking are about understanding and using numbers. The progressions of these deal with the basic operations, the kinds of quantitative relationships they model, and the kinds of problems they can be used to solve. Progressions in Operations and Algebraic Thinking initially begin with work with whole numbers, however the skills and concepts learned here can be applied to fractions, decimals, variables, and all number systems. The progression of Algebraic Thinking should be taught in conjunction with the basic operations. The skills in this progression will connect strongly and make use of the operations, but will also focus on building a strong Algebraic sense.

Algebraic Sense

K	• Connect conservation/cardinality of number to decomposition and composition of a number
	  ○ Recognize the cardinalities of small groups without having to count the objects – *perceptual subitizing,* e.g. recognizing the five dots on a die as "5" without counting
	  ○ Recognize that a collection of objects is composed of two subcollections and quickly combine their cardinalities to find the cardinality of the collection – *conceptual subitizing*, e.g. recognizing the 6 dots on a die as two rows of 3, and realizing the combined total is "6" without counting (**see Number Sense Progressions**)
	• Recognize mathematical symbols for addition (+), subtraction (-), and equal to/same as (=) as well as the terms *total* and *difference*
	• Represent simple addition and subtraction situations and models as expressions or equations (totals within 10)
	  ○ Match their models (physical and drawings) with equations or expressions provided by the teacher (cumulative skill for the year, following work with physical models and pictures)
	  ○ Work with problems situations including: *Add to – result unknown, Take from – result unknown, Put together/Take apart – total unknown,* and *Put together/Take apart – both addends unknown*
	  ○ Understand what a symbol in an equation represents before solving a problem (8 + 3 = □), and compare to the same equation after solving (8 + 3 = 11)
	  ○ Work with equations with operation symbols on either sign of the equal sign when appropriate (*take 8 apart to make 2 groups*: 8 = □ + ○)
	• Solve addition and subtraction equations for numbers within 5, while still connecting these equations to situations verbally or with drawings (**see Addition & Subtraction Progressions**)

1	• Extend work from kindergarten to match representations from problem situations to expressions or equations (totals within 20)
	  ○ Students begin to write equations to represent their work
	  ○ Problem situations include those from grade K in addition to Add to – change unknown, Add to – start unknown, Take from – change unknown, Take from – start unknown, Put together/take apart – addend unknown
	  ○ Extend to work with comparison situations: Compare – difference unknown, Compare – bigger unknown, Compare – smaller unknown
	  ○ Write equations to represent a problem that include a symbol (?, □, ○, etc.) for the unknown number
	  ○ Read to understand the problem situation, represent it directly with a situation equation, if necessary use a quantitative relationship to develop the solution equation, e.g. Represent a Take from – change unknown problem with the equation 14 - □ = 8, then realizing the 8 + □ = 14 can be better used to solve by thinking "how many more do I need to make 14"
	• Use equations to represent word problems that call for the addition of three whole numbers (with totals less than 20)
	• Understand the meaning of the equal sign
	  ○ Read "=" as *same as* rather than *equals*
	• Determine if equations involving addition and subtraction are true or false
	  ○ Justify their thinking, using concrete materials, pictures, or words to show that both sides of the equation show the same amount
	  ○ Write equations in all orientations, an operation on the right side and number on the left side of equation, an operation on the left side and number on the right side of equation, numbers on both sides of the equal sign, and operations on both sides of the equal sign (e.g. 6 = 6, 7 = 8 + 1, 5 + 2 = 2 + 5, 4 + 1 = 3 + 2, etc.)
	• Determine the unknown value in an addition or subtractions equation relating three whole numbers
	  ○ Solve for the unknown and explain how they found the unknown value in an equation
	  ○ Solve for the unknown in various positions in an equation, e.g. 8 + □ = 11, 5 = □ – 3, 6 + 6 = □, etc

2	• Extend work from grade 1 to solve one and two step word problems (within 100)

	○ Use drawings and equations with a symbol to represent the unknown number in a problem ○ Solve any of the problem structures ○ Extend thinking to solve two-step word problems ○ Use materials to represent the action(s) in the problem and connect that to a written equation ○ Determine the unknown in all positions using a variety of strategies, e.g. Part-Part-Whole, Number Line, Linking Cubes, etc. ○ Use the word *sum* as a synonym for *total* • Determine the unknown number in an equation relating four or more whole numbers, e.g. 37 + 10 + 10 = ___ + 17 • Determine whether a group of objects has an odd or even number of members (within 20) ○ Use concrete materials to pair objects ▪ Understand that the term *even* describes numbers divided into pairs with none left over ▪ Understand that the term *odd* describes numbers divided into pairs with one left over ○ Count objects or pictures by 2's to determine if the value is odd or even ○ Write equations to express an even number as a sum of two equal addends (doubles) ▪ Relate even and odd sets to doubles and doubles plus 1 facts ○ Begin to discover a pattern to determine if any number value is even or odd based on the digits in the number, i.e. looking only at the digit in the ones place can determine if a value is even or odd • Skip count by 5, 10, or 100 (**see Number Sense Progressions**) • Write an equation to express the total number of objects in an array as equal addends ○ Work with arrays with up to 5 rows and 5 columns ○ Given the number of rows and columns, model the arrays using physical objects or drawings on grid paper ○ Write repeated addition equations to represent the objects, i.e. 4 + 4 + 4 = 12 (**see Multiplication & Division Progressions**)
3	• Represent multiplication and division situations using equations and expressions ○ Use the symbols ●, ×, or * to represent equal group of objects ○ Use the symbols ÷, ⟌, or / to represent division into equal groups ○ In equations, use symbols to represent the unknown on both sides of the equation, e.g. ?, ___, □, or letters • Determine the unknown whole number in a multiplication or division equation relating three whole numbers, e.g. 8 x ? = 32, 5 = __ ÷ 3, 6 x 6 = □ ○ Write and read related multiplication equations with missing factors ○ Relate missing factor multiplication equations to division equations • Represent and solve two-step problems using equations with a letter standing for the unknown quantity ○ Connect to previous work with identifying missing information that was represented by a box, underscore, or other symbols ○ Include parentheses in both expressions and equations with more than one operation ○ Understand that operations inside the parentheses are to be done first ○ Understand that adding parentheses to equations/expressions can make computation easier ○ Apply the use of parentheses to work with the associative and distributive properties • Identify arithmetic patterns and explain them using properties or operations ○ Make generalizations about patterns as the foundation for algebraic reasoning ○ Look for and describe patterns in multiplication and addition tables ○ Justify and explain their reasoning • All above skills have important connections to **Multiplication & Division Progressions**
4	• Represent verbal statements of multiplicative comparisons as multiplication equations ○ Use a symbol for the unknown to represent the problem ○ Distinguish multiplicative comparison from additive comparison situations ○ Model and solve using various strategies including bar models (tape diagrams), concrete materials, pictures, words, and numbers • Represent multi-step problems with whole numbers, including those involving remainders, with an equation using a letter standing for the unknown quantity ○ Represent variables using a letter, based on the context of the word problem • Determine whether an equation is true or false by using comparative relational thinking

	○ Without performing the operation(s), determine if the values on each side of the equation are equal, e.g. justify that 60 + 24 = 57 + 27 *because 57 is 3 less than 60, but 27 is 3 more than 24* • Determine the unknown value in an equation relating four whole numbers by using comparative relational thinking ○ Without performing the operation(s), determine the unknown value, e.g. solve for *n*, 76 + 9 = *n* + 5 by thinking 9 is 4 more than 5, so *n* must be 4 less than 76, *n* = 72 ○ Addition and subtraction within 1000 ○ Multiplication limited to 1-digit x 2-digit, or a multiple of 10 x 1-digit ○ Division limited to 2-digit by 1-digit • Connect factors and multiples to make generalizations about prime and composite number values (working within range of numbers 1-100) ○ Extend the idea of multiplication and division facts and use patterns to determine the factors of a number ○ Extend the idea of multiplication, skip counting, and patterns to determine the multiples of a number ○ Identify and discuss patterns found when factoring a number, e.g. all even numbers have a factor of 2 ○ Distinguish between prime and composite numbers using patterns discovered with factors ▪ Number with only two factors are prime numbers ▪ Numbers with more than two factors are composite numbers ○ Determine if a number is a multiple of another given number • Given a rule, generate or extend a number or shape pattern ○ Model and record results of patterns using objects, pictures, numbers, tables, or words ○ Identify and describe features of the pattern ○ Make generalizations about the pattern, i.e. the numbers alternate between odd and even numbers ○ Work with number patterns involving all four operations, using only whole numbers 0 - 1000, and with no more than two operations in each given rule ○
5	• Use parentheses, brackets, and braces in numerical expressions ○ Evaluate expressions with these symbols ○ Solve problems and equations including parentheses (not including nested grouping symbols) ○ Solve problems and equations that employ order of operations (including the four operations and fractions with denominators of 10 or less, without division of fractions) ○ Explain their thinking as they use order of operations • Interpret expressions that record calculations with numbers ○ Interpret numerical expressions without evaluating them, e.g. understand 3 x (18932 + 921) is three times as large as 18932 + 921 without calculating (including the four operations and fractions with denominators of 10 or less, no division with fractions) ○ View expressions as entities created from component parts, i.e. (4 + 8) – (4 x 3) can be thought of as the difference between the sum of 4 + 8 and the product of 4 x 3 ○ Translate a mathematical expression in words to a written numerical expression and vice versa (multiplication symbol will be a cross, not the dot) • Extend work from 4th grade to generate two (whole number) numerical patterns using two given rules (not including rules with more than two operations) ○ Identify apparent relationships between corresponding terms • Form (whole number) ordered pairs consisting of terms from two patterns, e.g. record using a T-chart or table (in quadrant I) ○ Graph the ordered pairs on a coordinate plane (identify when a line connecting the ordered pairs is appropriate or not, is the data continuous?) • Describe patterns in tables, ordered pairs, and graphs • Graph ordered pairs in the first quadrant of the coordinate grid ○ Understand the first number indicates how far to travel from the origin in the direction of the x-axis and the second in the direction of the y-axis ○ Use the first quadrant of a coordinate grid to represent real-world problems