# MA.5.AR.2.2

**Overarching Standard:** ***MA.5.AR.2*** *Demonstrate an understanding of equality, the order of the operations and equivalent numerical expressions.*

**Benchmark of Focus**

MA.5.AR.2.2: Evaluate multi-step numerical expressions using order of operations.

*Examples:* Patti says the expression 12 ÷ 2 × 3 is equivalent to 18 because she works each operation from left to right. Gladys says the expression 12 ÷ 2 × 3 is equivalent to 2 because first multiplies 2 × 3 then divides 6 into 12. David says that Patti is correctlyusing order of operations and suggests that if parentheses were added, it would give more clarity.

Benchmark Clarifications

*Clarification 1:* Multi-step expressions are limited to any combination of arithmetic operations, includingparentheses, with whole numbers, decimals and fractions.

*Clarification 2:* Within this benchmark, the expectation is not to include exponents or nested groupingsymbols.

*Clarification 3:* Decimals are limited to hundredths. Expressions cannot include division of a fraction bya fraction.

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**Related Benchmark/Horizontal Alignment**

- MA.5.NSO.1.1/1.2/1.3/1.4/1.5
- MA.5.NSO.2.3/2.4/2.5/
- MA.5.FR.1.1/2.1

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**Vertical Alignment**

| Previous Benchmarks | Next Benchmarks |
|---|---|
| MA.4.AR.2.1 | MA.6.NSO.2.3 |
| MA.4.AR.2.2 | MA.6.AR.1.3 |

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**Terms from the K-12 Glossary**

- Expression
- Order of Operations

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**Purpose and Instructional Strategies**

The purpose of this benchmark is for students to use the order of operations to evaluate numerical expressions. In Grade 4, students had experience with numerical expressions involving all four operations (MA.4.AR.2.1/2.2), but the focus was not on order of operations. InGrade 6, students will be evaluating algebraic expressions using substitution and these expressions can include negative numbers (MA.6.AR.1.3).

- Begin instruction by exposing student to expressions that have two operations without any

grouping symbols, before introducing expressions with multiple operations. Use thesame digits, with the operations in a different order, and have students evaluate the expressions, then discuss why the value of the expression is different. For example, have students evaluate $6 \times 3 + 7$ and $6 + 3 \times 7$.

- In Grade 5, students should learn to first work to simplify within any parentheses, if present in the expression. Within the parentheses, the order of operations is followed. Next, while reading left to right, perform any multiplication and division in the order inwhich it appears. Finally, while reading from left to right, perform addition and subtraction in the order in which it appears.
- During instruction, students should be expected to explain how they used the order of operations to evaluate expressions and share with others. To address misconceptions around the order of operations, instruction should include reasoning and error analysistasks for students to complete (MTR.3.1, MTR.4.1, MTR.5.1).

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**Common Misconceptions or Errors**

- When students learn mnemonics like PEMDAS to perform the order of operations, theycan confuse that multiplication must always be performed before division, and likewiseaddition before subtraction. Students should have experiences solving expressions with multiple instances of procedural operations and their inverse, such as addition and subtraction, so they learn how to solve them left to right.

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**Strategies to Support Tiered Instruction**

- Instruction includes opportunities to solve expressions with multiple instances of procedural operations and their inverse, explicitly teaching the order of operations with an emphasis on the left to right order to solving multiplication and division, and addition and subtraction. Students use models or drawings as they solve.
    - For example, the teacher displays the following problem: $62 - 8 \times 4 + 3 - (18 \div 9)$. The teacher reviews the order of operations, reminding students that they must work to simplify within the parentheses first. The teacher then prompts students to multiply and divide from left to right next. Then, students are prompted to add and subtract from left to right and reminded that adding and subtracting fall within the same step. So, they will need to subtract $62 - 32$ to get 30 and then add $30 + 3$. The teacher repeats with additional expressions containing multiplication, division, addition, and subtraction in a variety of orders.

| | |
|---|---|
| Step 1: Parentheses | $62 - 8 \times 4 + 3 - (18 \div 9)$ |
| Step 2: Multiplication and division | $62 - 8 \times 4 + 3 - 2$ |
| Step 3: Addition and subtraction | $62 - 32 + 3 - 2$<br>$30 + 3 - 2$<br>$33 - 2$ |
| Solution | 31 |

- Instruction includes manipulatives to practice solving expressions with multiple instances of procedural operations and their inverse, such as addition and subtraction, so they learn how to solve them left to right. Instruction also includes explicitly teaching the order of operations with an emphasis on the left to right order to solving multiplication and division, and addition and subtraction. Students use manipulatives as they solve.
    - For example, display the following problem: 5 − 10 ÷ 5 + (2 × 3). The teacher reviews the order of operations, reminding students that they must work to simplify within the parentheses first. The teacher prompts students to multiply and divide from left to right next. Then, prompts students to add and subtract from left to right. Finally, the teacher reminds students that adding and subtracting falls within the same step, so they will need to subtract 5 − 2 before they add +6. This is repeated with additional expressions containing multiplication, division, addition, and subtraction in a variety of orders.

| | |
|---|---|
| Step 1:<br>Parentheses | 5 − 10 ÷ 5 + (2 × 3) |
| Step 2:<br>Multiplication and division | 5 − 10 ÷ 5 + 6 |
| Step 3:<br>Addition and subtraction | 5 − 2 +6<br>3 + 6 |
| Solution | 9 |

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**Questions to ask students:**

- **How can you use the order of operations to solve** 24 ÷ 6 × 2?
- Sample answer that indicates understanding: multiplication and division are interchangeable (because division is the opposite of multiplication) in order of operations, so we work left to right. In this example we should first divide 24 ÷ 6 = 4, then multiply 4 × 2 =8.
- Sample answer that indicates incomplete understanding or misconception: multiplication has to be done before division, (often students memorize the acronym PEMDAS or "My Dear Aunt Sally) so 6 × 2 = 12, then 24 ÷ 12 = 2.

- **How does changing the position of the parenthesis change the order in which the operations are solved?**
- Sample answer that indicates understanding: The operation that is in the parenthesis will always be solved first.

- **Write an example of a word problem that represents the expression 24 ÷ 6 × 2.**
- Sample answer that indicates understanding: There were 24 apples. They were separated equally into 6 different bags. The number of apples in each bag was then doubled. How many apples were there altogether?

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**Instructional Tasks**

*Instructional Task 1*

The two equations below are very similar. Are both equations true? Why or why not?

Equation One: 4 × 6 + 3 × 2 + 4 = 34

Equation Two: 4 × (6 + 3 × 2 + 4) = 64

*Instructional Task 2*

Part A. Insert one set of parentheses around two numbers in the expression below. Then evaluate the expression.

$$40 \div 5 \times 2 + 6$$

Part B. Now insert one set of parentheses around a different pair of numbers. Then evaluatethis expression.

$$40 \div 5 \times 2 + 6$$

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**Instructional Items**

*Instructional Item 1*

What is the value of the numerical expression below:

$(2.45 + 3.05) \div (7.15 - 2.15)$

*Instructional Item 2*

A numerical expression is evaluated as shown.

$\frac{1}{2} \times ( 3 \times 5 + 1) - 2$

In which step does the first mistake appear?

a. Step 1: $\frac{1}{2} \times (15 + 1) - 2$

b. Step 2: $\frac{1}{2} \times 14$

c. Step 3: $\frac{14}{3}$

d. Step 4: 7

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**Achievement Level Descriptors**

| Benchmark | Context | Assessment Limits |
|---|---|---|
| MA.5.AR.2.2 Evaluate multi-step numerical expressions using order of operations.<br>Example: Patti says the expression 12 ÷ 2 × 3 is equivalent to 18 because she works each operation from left to right. Gladys says the expression 12 ÷ 2 × 3 is equivalent to 2 because first multiplies 2 × 3 then divides 6 into 12. David says that Patti is correctly using order of operations and suggests that if parentheses were added, it would give more clarity.<br>Clarification 1: Multi-step expressions are limited to any combination of arithmetic operations, including parentheses, with whole numbers, decimals, and fractions.<br>Clarification 2: Within this benchmark, the expectation is not to include exponents or nested grouping symbols. | Mathematical | Items containing fractions will not include decimals. Items containing decimals will not include fractions. Expressions will not exceed three operations. Denominators will be limited to 1–10, 12, 16, 20, 50, and 100. |

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|---|---|---|---|
| Clarification 3: Decimals are limited to hundredths. Expressions cannot include division of a fraction by a fraction. | | | |
| **ALD 2** | **ALD 3** | **ALD 4** | **ALD 5** |
| Evaluates a two-step expression involving adding and subtraction using order of operations. | Evaluates multi-step expressions using order of operations but no use of parentheses | Evaluates multi-step numerical expressions using order of operations | Uses error analysis for determining whether a given evaluated expression includes an error at any given step in the evaluation process and evaluates multi-step numerical expressions using order of operations. |

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**Additional Resources:**
CPALMS Resources

Khan Academy: Order of Operations

Blog Post: A World Without Order (of Operations)

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**Resources/Tasks to Support Your Child at Home**:

Khan Academy: Evaluating Expressions with and without Parenthesis

Order of Operations Quiz